



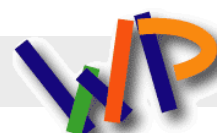
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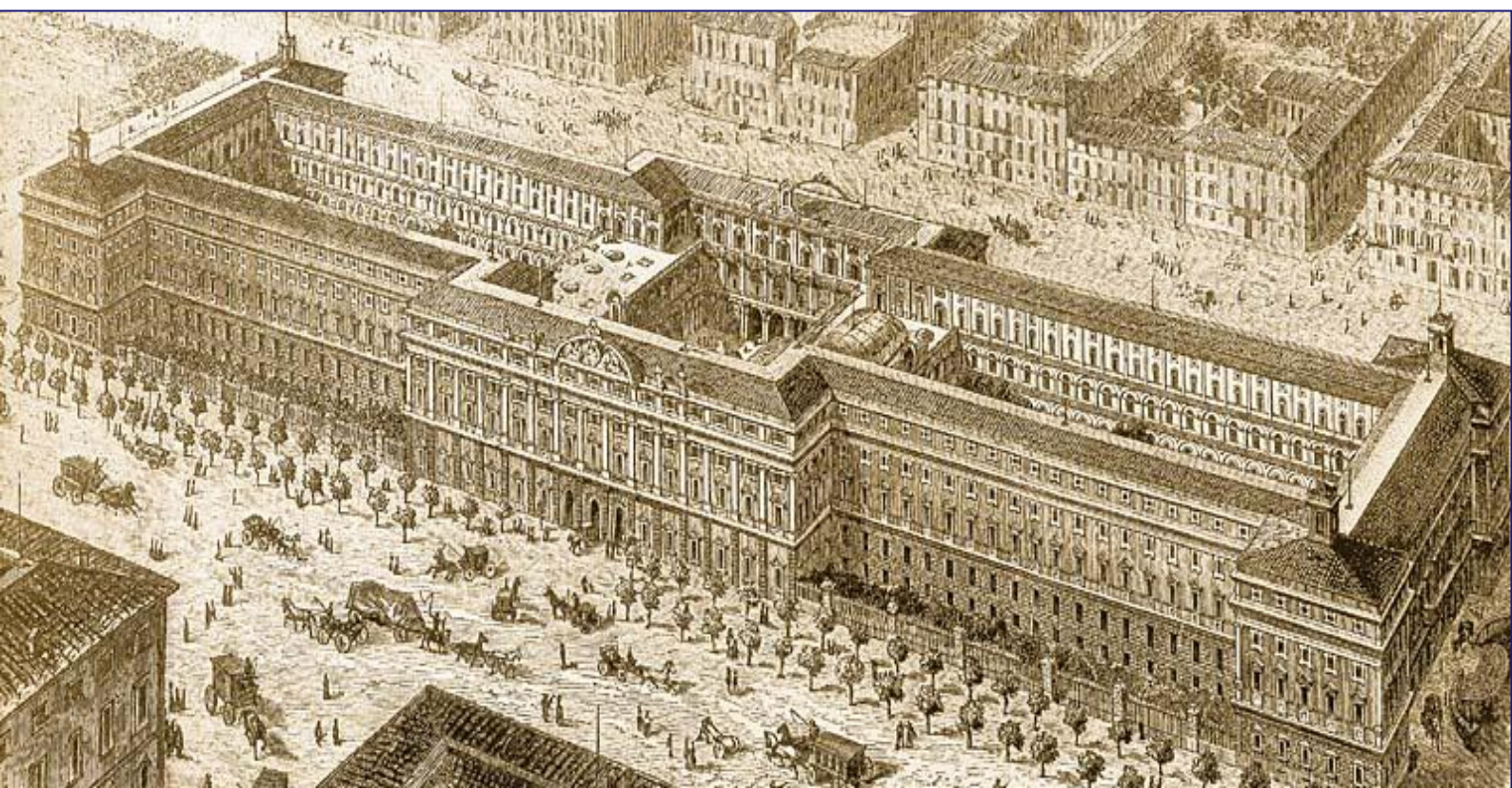
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Unemployment and Hysteresis: A Nonlinear Unobserved Components Approach

Silvestro DI SANZO, Alicia PEREZ-ALONSO



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Unemployment and Hysteresis: A Nonlinear Unobserved Components Approach¹

Silvestro DI SANZO*, Alicia PEREZ-ALONSO**

Abstract

A new test for hysteresis based on a nonlinear unobserved components model is proposed. Observed unemployment rates are decomposed into a natural rate component and a cyclical component. Threshold type nonlinearities are introduced by allowing past cyclical unemployment to have a different impact on the natural rate depending on the regime of the economy. The impact of lagged cyclical shocks on the current natural component is the measure of hysteresis. To derive an appropriate p-value for a test for hysteresis two alternative bootstrap algorithms are proposed: the first is valid under homoskedastic errors and the second allows for heteroskedasticity of unknown form. A Monte Carlo simulation study shows the good performance of both bootstrap algorithms. The bootstrap testing procedure is applied to data from Italy, France and the United States. We find evidence of hysteresis for all countries under study.

JEL Classification: C12, C13, C15, C32, E24

Keywords: Hysteresis; Unobserved Components Model; Threshold Autoregressive Models; Nuisance parameters; Bootstrap

(*) S. Di Sanzo, Confcommercio (the Italian General Confederation of Enterprises, Professional Occupations and Self-Employment), Research Department, Piazza G. G. Belli 2, 00153 Roma, Italy. E-mail: s.disanzo@confcommercio.it.

(**) A. Perez-Alonso (corresponding author): Research Group in Economic Analysis, Facultad de Ciencias Económicas e Empresariales, Universidade de Vigo, Campus As Lagoas-Marcosende s/n, 36310 Vigo, Pontevedra, Spain. Phone: +34986812502. Fax: +34986812402. E-mail: apereza@uvigo.es;

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1 INTRODUCTION

In the mid 1970's European unemployment started a transition from rates in the order of 1-2% to rates in the order of 10-15% in the 1990's. More recently, according to Eurostat, the euro area seasonally-adjusted unemployment rate stood at 7.5% in September 2008. This experience reveals a slow tendency of actual unemployment to revert to a stable underlying unemployment rate, if any. Many theories have emerged to provide an economic explanation which could account for this observed unemployment persistence. Most of the work in the relevant literature assumes that it can be attributed to changes in the natural rate of unemployment and/or changes in the cyclical rate of unemployment. Based on this framework, two main approaches are the natural rate theory and the unemployment hysteresis theory.

The first approach assumes that output fluctuations generate cyclical movements in the unemployment rate, which in the long run, will tend to revert to its equilibrium. The crux of the natural rate hypothesis is that the cyclical unemployment rate and the natural unemployment rate evolve independently. Hence, the tendency of the natural rate to remain at a high level is the result of permanent shocks on the structure of the labour market, whereas transitory shocks only cause a temporary deviation from a unique equilibrium, see Friedman (1968), Bean et al. (1987) and Layard et al. (1991).

The second approach assumes that the cyclical unemployment rate and the natural unemployment rate do not evolve independently. The basic idea is that a change in the cyclical component of the unemployment rate may be permanently propagated to the natural rate (see Amable et al. 1995 and Roed 1997 for a survey). Therefore, a direct implication of the hysteresis hypothesis is that short-run adjustments of the economy can take place over a very long period. Consequently, aggregate demand policy, traditionally considered as ineffective in changing the natural rate of unemployment, can have a permanent effect on it.

In this paper, we focus on the second approach. The concept of hysteresis is brought to the forefront of labour market theory through a paper by Blanchard and Summers (1986). They consider an insider-outsider model of wage bargaining between insiders and the firm with outsiders playing no role². Given the presence of labour turnover costs, a shock that reduces the number of insiders one period raises the optimal insider-wage in subsequent periods, which prevents unemployed workers from being hired. In the particular case where insider status always coincides with current employment, employment follows a random walk. Based on this framework, a great number of empirical studies have investigated whether unemployment series, which is mainly modelled as a linear ARMA-type process, exhibits a unit root (see Roed 1997 and references therein). However, this practice of checking for the presence of hysteresis using linear ARMA-type processes has an important shortcoming: natural and cyclical shocks are summarized in the innovation with no distinction. Given that hysteresis in unemployment arises when a change in cyclical unemployment induces a permanent change in the natural rate, the presence of a unit root in the unemployment rate, modelled as a linear dynamic system, could be generated by accumulation of natural shocks and be completely independent of

²For further details on the insider-outsider theory of employment see Lindbeck and Snower (1988).

whether there is hysteresis. Hence, separating the respective effects of transitory and permanent shocks on the natural rate of unemployment is the only way to assess if changes in it are due to cyclical (this is the case of hysteresis) or natural shocks or both. So, we need an econometric model that discriminates between natural and cyclical sources of influence on the unemployment rate.

Jaeger and Parkinson (1994, henceforth JP) put this idea into perspective and adopt an unobserved components model to test the validity of the hysteresis hypothesis³. They generate a pure statistical decomposition of the actual unemployment rate into a natural rate component and a cyclical component, which are both treated as latent variables. They also assume a particular structure to describe the variation over time of these latent variables. The hysteresis effect is introduced by allowing cyclical unemployment to have a lagged effect on the natural rate, which is assumed to contain a unit root. They only consider symmetric responses of the natural rate as regards cyclical unemployment fluctuations. Thus, they implicitly assume hysteresis is a linear phenomenon, and this assumption may be too restrictive.

We propose an extended version of JP's model introducing nonlinearities. There is a wide range of theoretical and empirical evidence that shows that the unemployment rate displays asymmetries in adjustment dynamics, and thus hysteresis may be characterized by nonlinear dynamics when it exists. Following, we look at some of the various explanations for the nonlinearity of the unemployment rate. Firstly, there are asymmetric adjustment labour costs, such as hiring and firing costs (see Johansen 1982, Bentolila and Bertola 1990, and Hamermesh and Pfann 1996). Secondly, there is asymmetry in job creation and destruction, for instance, Mortensen and Pissarides (1993) emphasize the time it takes for a firm to find a good match to explain why jobcreation takes longer than job destruction. Similarly, Caballero and Hammour (1994) develop a model in which jobs are destroyed at a higher rate during recessions than expansions. Finally, Bean (1989) stresses asymmetry in capital destruction. The theoretical arguments stressing the nonlinearity of unemployment have been matched by plenty of empirical evidence. Using nonparametric techniques, the seminal paper of Neftci (1984) finds unemployment rises to be sudden, and falls to be gradual; see also Sichel (1989) and Rothman (1991). Various parametric nonlinear time series models of unemployment have also been estimated in the literature by Hansen (1997), Bianchi and Zoega (1998), Koop and Potter (1999), Papell et al. (2000), Caner and Hansen (2001), Skalin and Teräsvirta (2002), Coakley and Fuertes (2006), Caporale and Gil-Alana (2007), among others. All these studies assume Markov-switching, threshold or smooth transition specifications.

This theoretical and empirical evidence suggests that any satisfactory model for the unemployment rate has to be able to account for nonlinearity. The contribution of this paper is to extend JP's model by introducing nonlinearities using a threshold autoregressive (TAR) model⁴. In particular, we allow past cyclical unemployment to have a different effect on the natural rate, which depends on the regime of the economy. We consider two regimes reflecting favorable and unfavorable times, which have been defined based on previous changes in unemployment. We choose this particular form of nonlinearity because TAR models are the most widely used class of models in the nonlinear time series literature on the dynamics of unemployment, given

³See Harvey (1989) for a detailed description of the Unobserved Component models.

⁴For an extensive discussion of TAR models we refer to Tong (1990).

that they can exhibit the type of dynamic asymmetries that theoretical models suggest, and are computationally easy to work with (see references above). Furthermore, Petrucci (1992) shows that threshold specifications may be viewed as an approximation to a more general class of nonlinear models. We propose a test for assessing the presence of regime specific nonlinearity within the phenomenon of hysteresis when it exists. The relevant null hypothesis is a one-regime model against the alternative of two regimes, i.e. the null hypothesis of linearity is tested against a threshold alternative. Testing for threshold type nonlinearities raises a particular problem known in the statistics literature as hypothesis testing when a nuisance parameter is not identified under the null hypothesis (see, Davies 1977 and 1987, Andrews and Ploberger 1994, Chan 1990, and Hansen 1996). If the model is not identified under the null, the asymptotic distribution of classical tests is unknown, so tabulated critical values are unavailable. To circumvent this problem, we use bootstrap methods to approximate the null distribution of the test statistic. More precisely, we use the resampling algorithm proposed by Stoffer and Wall (1991) for linear state-space models. Finally, we use this bootstrap testing procedure to check for the presence of hysteresis in Italy, France and the United States.

The rest of this paper is organized as follows. Section 2 briefly describes JP's model and proposes an extended version that introduces hysteresis allowing for threshold type nonlinearity. Section 3 proposes two alternative bootstrap procedures to compute the p-value for a linearity test under our hysteresis model. Empirical results for Italy, France and the United States are presented in Section 4. The conclusion is provided in the last section. Appendix A discusses the design of the Monte Carlo experiments that are used to investigate the small sample performance of the bootstrap version of the test statistic, and presents the results of some limited simulations. Estimation methods are relegated to Appendix B. Appendix C contains all the tables and figures.

2 AN EXTENSION OF JAEGER AND PARKINSON'S MODEL

JP propose a pure statistical decomposition of the unemployment rate to evaluate the data for evidence on hysteresis effects. They assume the actual unemployment rate to be the sum of two unobservable components: a non-stationary natural rate component, U^N , and a stationary cyclical component, U^C ,

$$(2.1) \quad U_t = U_t^N + U_t^C.$$

The natural rate component is defined as a random walk plus a term capturing possible hysteresis effects,

$$(2.2) \quad U_t^N = U_{t-1}^N + \alpha U_{t-1}^C + \epsilon_t^N.$$

Coefficient α measures, in percentage points, how much the natural rate increases if the economy experiences a cyclical unemployment rate of 1.0 percent. The size of this coefficient is their measure of hysteresis.

The cyclical component of the unemployment rate is defined as a stationary second-order

autoregressive process⁵,

$$(2.3) \quad U_t^C = \phi_1 U_{t-1}^C + \phi_2 U_{t-2}^C + \epsilon_t^C.$$

To identify the model, the system is completed by augmenting it with a version of Okun's law, which relates cyclical unemployment and output growth,

$$(2.4) \quad D_t = \beta D_{t-1} + \delta U_t^C + \epsilon_t^D,$$

where D_t stands for the output growth rate at date t ⁶. Equation (2.4) defines the output growth rate as an autoregressive process of order one plus a term capturing the influence of the cyclical rate of unemployment. Since the cyclical component is assumed to be stationary, we consider U_t^C instead of ΔU_t^C as in JP's model in order to avoid a problem of over-differentiation.

The disturbances ϵ_t^N , ϵ_t^C and ϵ_t^D are assumed to be mutually uncorrelated shocks, which are normally distributed with variances σ_N^2 , σ_C^2 and σ_D^2 , respectively.

To test the hysteresis hypothesis, i.e. past cyclical movements on unemployment have a permanent impact on the natural rate, JP perform a significance test on parameter α ,

$$H_0^{JP} : \alpha = 0 \quad \text{vs.} \quad H_1^{JP} : \alpha \neq 0.$$

If parameter α is significantly different from zero, they argue there exists a hysteresis effect on the unemployment rate. Note that JP's model is linear given that past cyclical unemployment changes have the same impact, in absolute terms, on the natural unemployment rate. For example, a variation in the cyclical component of 1% or (-1)% causes a variation in the natural rate of $\alpha\%$ or $(-\alpha)\%$, respectively.

Relaxing the linearity assumption may allow a better estimation of hysteresis if it exists. It is widely acknowledged that the unemployment rate displays asymmetries in adjustment dynamics. In particular, fast-up, slow-down dynamics. As pointed out in the introduction, among the multitude of alternative nonlinear models available, we choose the class of models with TAR dynamics. Hence, to relax the assumption of linearity, we introduce threshold type nonlinearities into JP's model. These are introduced by allowing past cyclical unemployment to have a different impact on the natural rate, which depends on the regime of the economy. To that end, equation (2.2) becomes

$$(2.2') \quad U_t^N = U_{t-1}^N + \alpha_1 U_{t-1}^C I(q_{t-1} \geq \gamma) + \alpha_2 U_{t-1}^C I(q_{t-1} < \gamma) + \epsilon_t^N,$$

where q_{t-1} is the threshold variable assumed to be stationary, γ stands for the threshold parameter and $I(B)$ is the usual indicator function taking the value one when B holds and zero otherwise. Equations (2.1), (2.3) and (2.4) remain the same together with assumptions about shocks.

This model is estimated via maximum likelihood (ML) in the framework of the Kalman filter⁷. More precisely, we employ a modified Kalman filter to incorporate a deterministic cut-off of the sample that corresponds to a raw indicator for *favorable* and *unfavorable* periods, which is based on the methodology implemented for the estimation of TAR models. We choose the long

⁵To select the order of the autoregressive process, the Akaike and Schwarz information criteria and the diagnostic checking tests proposed by Harvey (1985) are employed. As in JP, we find that an AR(2) process for the cyclical component fits the data well for all the countries under study.

⁶Our results regarding the nature of the hysteresis phenomenon are rather stable even when the model is estimated using different identification equations such as the Phillips curve and the Beveridge curve.

⁷See Appendix A for a detailed description of this estimation methodology.

difference $U_{t-1} - U_{t-d}$ with $d \in \{2, 3\}$ as our threshold variable q_{t-1} . This variable is an indicator of the state of the economy to identify the regimes. The integer d is called the threshold delay lag. Whether the threshold variable is lower or higher than the threshold parameter γ determines whether an observation belongs to one regime or the other. We consider an economy with two regimes, one related to high long differences (regime 1), i.e. an *unfavorable* regime, and the other with low long differences (regime 2), i.e. a *favorable* regime. Parameters d and γ are unknown so they are estimated along with the other parameters of the model. The maximization is best solved through a grid search over the two-dimensional space (γ, d) . To execute a grid search we need to fix a region over which to search. It is important to restrict the set of threshold candidates *a priori* so that each regime contains a minimal number of observations. For each value of d , we restrict the search to values of γ lying on $[\underline{\gamma}, \bar{\gamma}]$, where $\underline{\gamma}$ is the τ th quantile of q_{t-1} , and $\bar{\gamma}$ is the $(1 - \tau)$ th quantile. In our applications we choose $\tau = 0.30$. Then we estimate the model for each pair (γ, d) belonging to the grid $\Delta = ([\underline{\gamma}, \bar{\gamma}] \times \{2, 3\})$ and retain the one that provides the highest log-likelihood value.

In this framework, we want to test the null hypothesis of a linear model versus the alternative of a nonlinear one, that is:

$$H_0 : \alpha_1 = \alpha_2 \quad \text{vs.} \quad H_1 : \alpha_1 \neq \alpha_2.$$

If we reject H_0 (the null of linearity), there is evidence for the presence of hysteresis in the unemployment rate, which displays a nonlinear behaviour. This finding is consistent with cyclical shocks being propagated asymmetrically to the natural rate. In this case, JP's model is misspecified and any inference based on the parameters of their model may lead us to wrong conclusions. If it is not rejected, the next step is to estimate the linear model proposed by JP and test for hysteresis following the strategy they propose. If we reject H_0^{JP} , the natural rate component is affected by movements in the cyclical component and thus hysteresis in unemployment occurs. If it is not rejected, there is no place for hysteresis.

Here we propose a Wald type test statistic for testing H_0 . Note that under this null hypothesis the threshold parameter given by γ and the delay d remain unidentified. As a result, the asymptotic distribution of conventional test statistics is not χ^2 . This is a well-known problem in the literature on testing for regime switching type of nonlinearities; here we test for a single regime against the alternative of two regimes. This problem is usually handled by viewing the test statistic as a random function of the nuisance parameters and basing inference on a particular functional of the test statistic such as, for instance, its supremum over (γ, d) (see, Davies 1977 and 1987, Andrews and Ploberger 1994, Chan 1990, and Hansen 1996). Letting $W(\gamma, d)$ denote the Wald type test statistic obtained for each (γ, d) , we base our inferences on $\text{Sup}W = \sup_{(\gamma, d) \in \Delta} W(\gamma, d)$. To our knowledge the null asymptotic distribution of $\text{Sup}W$ is unknown under the above framework. To circumvent this problem, we suggest using bootstrap methods to approximate the sampling distribution of $\text{Sup}W$ under H_0 .

3 TESTING FOR LINEARITY

As the asymptotic distribution of the SupW test statistic is unknown in the present framework, in this section we discuss two bootstrap methods to calculate p -values. As a general rule, resampling should always reflect the null hypothesis, according to Hall and Wilson (1991). Under the null hypothesis of linearity we have JP's model, and Stoffer and Wall (1991) establish the validity of a resampling scheme for the innovations sequence of linear state-space models. Other work using the bootstrap to study the problem of testing for linearity includes Hansen (1999), Caner and Hansen (2001) and Hansen and Seo (2002).

To approximate the sampling distribution of the SupW test statistic, we suggest using either a parametric residual bootstrap, or alternatively a wild bootstrap. The parametric residual bootstrap requires a complete specification of the model under H_0 . This is JP's model but relaxing the strong assumption that the error terms are normally distributed. While the assumptions of the model also include homoskedasticity, we do not think that it is prudent to impose this condition when constructing test statistics. It is therefore desirable to calculate a bootstrap distribution of SupW allowing for the possibility of error terms with an unknown pattern of heteroskedasticity. The disadvantage of the parametric residual bootstrap is that if the pattern is unknown, it cannot be imitated in the bootstrap data-generating process under H_0 . A technique used to overcome this difficulty is the so-called wild bootstrap proposed by Wu (1986) and developed by Liu (1988).

The finite sample performance of the test statistic obtained from the two bootstrap algorithms is investigated with Monte Carlo experiments in Appendix A. The simulation results suggest that the bootstrap test statistic works quite well concerning size and power in our framework. Of course, we have no guarantee that it works in general.

3.1 The state-space model

The state-space model is defined by the equations

$$(3.1) \quad s_t = F(q_{t-1})s_{t-1} + w_t,$$

$$(3.2) \quad y_t = Hs_t + Dy_{t-1} + v_t,$$

where $s_t = (U_t^N, U_t^C, U_{t-1}^C)'$ is a vector of unobserved state variables and $y_t = (U_t, D_t)'$ is a vector of observed variables. Equation (3.1) is known as the transition equation and equation (3.2) is known as the measurement equation. The coefficients of the model are stored in the constant matrices

$$F(q_{t-1}) = F_1 I(q_{t-1} \geq \gamma) + F_2 I(q_{t-1} < \gamma), \quad q_{t-1} = U_{t-1} - U_{t-d}, \quad d \in \{2, 3\};$$

$$H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & \delta & 0 \end{bmatrix}; \quad F_i = \begin{bmatrix} 1 & \alpha_i & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix}, \quad i = 1, 2; \quad D = \begin{bmatrix} 0 & 0 \\ 0 & \beta \end{bmatrix}.$$

The vectors $w_t = (\epsilon_t^N, \epsilon_t^C, 0)'$ and $v_t = (0, \epsilon_t^D)'$ represent white noise processes with $E(w_t w_t') = Q$, $E(v_t v_t') = R$ and $E(w_t v_t') = 0$, where

$$Q = \begin{bmatrix} \sigma_N^2 & 0 & 0 \\ 0 & \sigma_C^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} 0 & 0 \\ 0 & \sigma_D^2 \end{bmatrix}.$$

Note that under the null hypothesis of linearity $F(q_{t-1}) = F$. To simplify the notation, let $\theta_0 = (\sigma_N, \sigma_C, \sigma_D, \alpha, \phi_1, \phi_2, \delta, \beta)'$ be the vector with the model coefficients and the correlation structure under H_0 , and $\theta_1 = (\sigma_N, \sigma_C, \sigma_D, \alpha_1, \alpha_2, \phi_1, \phi_2, \delta, \beta)'$ be the vector of parameters under the alternative of nonlinearity.

3.2 Two bootstrap algorithms

The first algorithm we propose is the parametric residual bootstrap (RB). It consists of the following steps:

Bootstrap I (RB)

Step 1

We compute the SupW test statistic. To compute it we need only to estimate the model under H_1 . For each given value of $(\gamma, d) \in \Delta$, let $\hat{\theta}_1(\gamma, d)$ denote the ML estimate of θ_1 . We compute the pointwise Wald test statistic as $W(\gamma, d) = R\hat{\theta}_1(\gamma, d) \left(R\widehat{\text{Var}}(\hat{\theta}_1(\gamma, d))R' \right)^{-1} (R\hat{\theta}_1(\gamma, d))'$, where R is the selector matrix $R = (0 \ 0 \ 0 \ 1 - 1 \ 0 \ 0 \ 0 \ 0)$ and $\widehat{\text{Var}}(\hat{\theta}_1(\gamma, d))$ is the robust variance-covariance matrix estimator proposed by White (1982). Davies (1977, 1987) suggest testing H_0 by $\text{Sup}W = \sup_{(\gamma, d) \in \Delta} W(\gamma, d)$.

Step 2

We use the Kalmanfilter to construct the standardized residuals under H_0 . We first obtain linear forecasts of the state vector at time t based on all the available information up to time $t - 1$, say $s_{t|t-1}$, and the mean square error matrix associated with each of these forecasts, say $P_{t|t-1}$. We also obtain from the Kalmanfilter the *innovations* $\epsilon_t = y_t - Hs_{t|t-1} - Dy_{t-1}$, the *innovations covariance matrix* $\Sigma_t = HP_{t|t-1}H' + R$, the *Kalman gain matrix* $K_t = P_{t|t-1}H'\Sigma_t^{-1}$, and the *updating of the state variable* $s_{t|t} = s_{t|t-1} + K_t\epsilon_t$. We also derive the *innovations form representation* of the observations as

$$(3.3) \quad s_{t+1|t} = Fs_{t|t-1} + FK_t\epsilon_t,$$

$$(3.4) \quad y_t = Hs_{t|t-1} + Dy_{t-1} + \epsilon_t.$$

Let $\hat{\theta}_0$ denote the ML estimate of θ_0 . Evaluating $\epsilon_t, \Sigma_t, K_t$ and $s_{t|t}$ at $\hat{\theta}_0$, we obtain $\hat{\epsilon}_t, \hat{\Sigma}_t, \hat{K}_t$ and $\hat{s}_{t|t}$. We construct the standardized residuals by setting $e_t = \hat{\Sigma}_t^{-1/2}\hat{\epsilon}_t$. By using standardized residuals, we guarantee that all model residuals have, at least, the same first two moments.

Step 3

The bootstrap errors $\{e_t^*, t = 1, \dots, T\}$ are independent values obtained by resampling, with replacement, from the set of standardized residuals $\{e_t, t = 1, \dots, T\}$.

Step 4

To construct the bootstrap data set under H_0 , say $\{y_t^*, t = 1, \dots, T\}$ we use equations (3.3) and (3.4). Let \hat{F}, \hat{H} and \hat{D} be the matrices of coefficients evaluated at $\hat{\theta}_0$, and $s_{1|0} = 0$ contains the first 3 values of the state variables (thus, these are prespecified and set equal to the initial

conditions for the Kalman filter). The remaining elements of the vectors $s_{t+1|t}$ are constructed by computing a first-order autoregressive process given by (3.3):

$$s_{t+1|t}^* = \hat{F}s_{t|t-1}^* + \hat{F}\hat{K}_t\hat{\Sigma}_t^{1/2}e_t^*.$$

The vector y_t is constructed by computing a first-order autoregressive process, with initial conditions fixed at the observed values, and then by adding the results to the corresponding elements of $s_{t+1|t}$. That is, the row t th of y is given by (3.4):

$$y_t^* = \hat{H}s_{t|t-1}^* + \hat{D}y_{t-1}^* + \hat{\Sigma}_t^{1/2}e_t^*.$$

All initial conditions are kept fixed throughout the bootstrap replications.

Step 5

The bootstrap sample $\{y_t^*, t = 1, \dots, T\}$ is then used to calculate the statistic SupW^* using the same procedure as to calculate SupW on the actual series.

Step 6

Repeating steps 3 through 5 for $b = 1, \dots, B$, gives a sample $\{\text{SupW}^*: b = 1, \dots, B\}$ of SupW values. This sample mimics a random sample of draws of SupW under H_0 . We compute the bootstrap p -value as $p_B = \text{card}(\text{SupW}^* \geq \text{SupW})/B$, that is the fraction of SupW^* values that are greater than the observed value SupW . We carry out $B = 1000$ bootstrap replications.

The wild bootstrap (WB) is an alternative way of obtaining the bootstrap distribution of the SupW test statistic allowing for the possibility of heteroskedasticity of unknown form. This bootstrap algorithm differs from the former in the resampling scheme of the residuals and in the use of a (conditionally) fixed design on the regressors to obtain the bootstrap data set.

Bootstrap II (WB)

Step 3'

To construct the wild bootstrap errors $\{\tilde{e}_t^*, t = 1, \dots, T\}$, we first generate η_t independent and identically distributed random variables from a fixed distribution, such that $E(\eta_t) = 0$ and $E(\eta_t^2) = E(\eta_t^3) = 1$.⁸ We next define $\tilde{e}_t^* = \hat{e}_t\eta_t$, where \hat{e}_t is the t th non-standardized residual calculated in step 2. Thus, the errors \tilde{e}_t^* satisfy $E^*(\tilde{e}_t^*) = 0$, $E^*(\tilde{e}_t^{*2}) = \hat{e}_t^2$ and $E^*(\tilde{e}_t^{*3}) = \hat{e}_t^3$, where $E^*(\cdot)$ denotes the expectation under the bootstrap distribution.

Step 4'

To construct the bootstrap data set $\{\tilde{y}_t^*, t = 1, \dots, T\}$:

- I. Set the initial condition $s_{1|0} = 0$ and, for $t = 2, \dots, T$, set $\tilde{s}_{t|t-1}^* = \hat{s}_{t|t-1}$, that is, unobserved bootstrap components are generated with conditionally set design on the estimated unobserved components in step 2: $\tilde{s}_{t+1|t}^* = \hat{F}\hat{s}_{t|t-1} + \hat{F}\hat{K}_t\tilde{e}_t^*$.
- II. Using a conditional resampling on $(y_0, y_1, \dots, y_{T-1})$, derive $\tilde{y}_t^* = \hat{H}\tilde{s}_{t|t-1}^* + \hat{D}y_{t-1} + \tilde{e}_t^*$, $t = 1, \dots, T$.

⁸In particular, the variable η_t was sampled from Mammen's (1993, p.257) two-point distribution attaching masses $(5 + \sqrt{5})/10$ and $(5 - \sqrt{5})/10$ at the points $(\sqrt{5} - 1)/2$ and $(\sqrt{5} + 1)/2$, respectively.

4 EMPIRICAL RESULTS

Our study concerns Italy, France and the United States. The economic series employed are the quarterly unemployment rate (U) and real gross domestic product (GDP). Data for Italy (1970:1-2007:2) come from *Prometeia*, and data for France (1978:1-2007:2) and U.S. (1965:1-2007:2) come from *OECD Main Economic Indicators*. All data are obtained as seasonally adjusted and all the variables except the unemployment rate are in natural logs.

We have decomposed the unemployment rate assuming that the natural rate contains a unit root. This assumption must be tested. We employ the methodology proposed by Caner and Hansen (2001) to test for a unit root in a single-equation two-regime TAR model. They restrict their analysis to univariate time series. Therefore, we adapt their method to our framework of state-space models by mimicking the method. We obtain that the unemployment rate series displays a non-stationary behaviour for all countries. We perform an augmented Dickey-Fuller (ADF) unit root test for the GDP series, which also displays a non-stationary behaviour for all countries. Results are presented in Table 1.

Tests for hysteresis are reported in Table 2. The p -values presented in Table 2 are calculated following the bootstrap technique described in Section 3. For comparison reasons, we also report the p -values obtained with the linear model of JP. Diagnosis checking of the residuals of the linear model leads us to implement a wild bootstrap for the U.S. and a parametric residual bootstrap for France and Italy. According to bootstrap p -values, the hysteresis effect is significant at the 1% level for all countries. As argued in Section 2, under the presence of nonlinearity, JP's model may lead to obtain spurious inference results. In fact, note that JP's methodology fails to detect hysteresis for the case of Italy, France and U.S..

Results concerning the estimated models for Italy, France and U.S. are available in Table 3. For the case of Italy, the ML estimate of the threshold parameter is $\hat{\gamma} = 0.1$ with a 90% bootstrap confidence interval [0:023; 0:247]. Our estimate of the delay parameter is $\hat{\delta} = 2$. Hence, the threshold model splits the regression into two regimes depending on whether or not the threshold variable is higher than this threshold parameter. That is, we consider we are in regime 1 when $U_{t-1} - U_{t-2} \geq 0.1$ and in regime 2 when $U_{t-1} - U_{t-2} < 0.1$.

For Italy, there are less observations in regime 1 (41%) than in regime 2 (59%), which means that this country spent more periods of time in the *favorable* regime. This is also the case for U.S. and France. Analyzing the estimated hysteresis parameter, we observe a point of great interest. Both parameters are positive and the one associated with Regime 1 is greater than that of Regime 2. This points to asymmetric responses of the natural rate as regards cyclical unemployment movements in the following direction: the natural rate does not decrease in *favorable* cyclical periods as much as it increases in *unfavorable* cyclical periods. The size of the coefficients suggests that this mechanism is more pronounced in France than in U.S. and Italy. In fact, for Italy, the natural rate decreases (2.512%) in *unfavorable* periods, while cyclical shocks have an impact of (1.476%) in *favorable* periods. In the U.S., these values are (1.343%) and (0.562%), respectively. On the other hand, for France, we find (3.540%) and (1.570%), respectively.

In Figures 1-3, the estimate of the natural rate is depicted against the recessionary periods for each country. Apart from the U.S., for which the *NBER Business Cycle Dating Committee* has been dating expansion and recessions, which have been generally recognized as the official U.S. business cycle dates, there is no widely accepted reference chronology of the

classical business cycle for other countries. To overcome this problem, we date the turning points by using the dating algorithm of Harding and Pagan (2002) that isolates the local minima and maxima in a quarterly series, subject to reasonable constraints on both the length and amplitude of expansions and contractions. For the U.S., periods of increasing natural rate correspond to, but generally lag, the NBER recession periods. This is consistent with the classification of the unemployment rate as a lagging indicator at troughs.

Our findings have three important implications. Firstly, our empirical evidence supports theoretical models of hysteresis that describe it as a nonlinear phenomenon (see Bentolila and Bertola 1990, and Caballero and Hammour 1994 among others). As we pointed out in the introduction, in these models, hysteresis arises when cyclical shocks are propagated asymmetrically to the natural rate. Secondly, since statistical linear models are not able to describe the dynamic asymmetries of the unemployment rate, nonlinear models are needed to correctly represent and test hysteresis phenomena. Here, JP's hysteresis test may lead to obtain misleading inference results. Thirdly, our results are important for policy-makers. When hysteresis is present in the labour market, monetary policies, traditionally considered as ineffective, can be used to combat unemployment without immediate inflationary consequences. This evidence is in contrast with non-accelerating inflation rate of unemployment (NAIRU) models where shocks are not long-lived, and thus the unemployment rate reverts back to its underlying equilibrium level (see Friedman 1968).

5 CONCLUSIONS

In this paper we propose a new test for hysteresis based on a nonlinear unobserved components model. We extend the model of Jaeger and Parkinson (1994) by introducing threshold type nonlinearities into the specification of the natural rate component. We do this by allowing past cyclical unemployment to have a different effect on the current natural rate depending on the regime of the economy. Under this framework, a test on the hysteresis parameter implies to perform a test for linearity. In particular, the null hypothesis of interest is that of a one-regime model versus the alternative of two regimes. Testing for the presence of a threshold effect involves nuisance parameters which are not identified under the null hypothesis of linearity, so the asymptotic distribution of standard tests is unknown under the null, precluding tabulation of critical values. We rely on bootstrapping techniques to calculate an appropriate p -value for the test statistic. In particular, we propose two bootstrap procedures: the first is valid if the errors are homoskedastic and the second allows for general forms of heteroskedasticity. To assess the usefulness of the bootstrap test for linearity, finite sample results are reported in a simple Monte Carlo study. Our study concerns Italy, France and the United States. The empirical results show that the presence of hysteresis cannot be rejected for all the countries under study.

A. MONTE CARLO EVIDENCE

In this section we report on a Monte Carlo simulation study designed to evaluate the small sample performance of both bootstrap procedures in the problem of testing for linearity. We start with a brief description of the design of the experiment, then proceed with the discussion of the results.

Design of the experiment

The time series considered in our analysis are generated according to the state-space model given by equations (3.1) and (3.2), under the null and the alternative hypotheses. Let M_0 and M_1 denote the class of linear and nonlinear state-space models, respectively. Thus, in our experiments we use M_0 and M_1 as data-generating processes (DGPs) with $(\epsilon_t^N, \epsilon_t^C, \epsilon_t^D)' \text{ iid } N(0, \Omega)$, where

$$\Omega = \begin{bmatrix} \sigma_N^2 & 0 & 0 \\ 0 & \sigma_C^2 & 0 \\ 0 & 0 & \sigma_D^2 \end{bmatrix}.$$

We aim at testing the null hypothesis of linearity. As discussed at the end of Section 2, the null hypothesis is true if and only if $\alpha_1 = \alpha_2$. Hence, M_0 is nested in M_1 . We use the statistic SupW based on an estimated M_1 setting $d = 2$, and compute the p -value using both the residual bootstrap and the wild bootstrap. The size of the test is investigated when the data are generated according to M_0 , while turning to the power properties of the test under M_1 .

To ensure the relevance of the simulations, the parameter values are chosen to correspond to models that have been fitted successfully to real-world time series. More specifically, we choose the estimated parameters for the U.S. obtained by JP as the DGP under the null hypothesis (DGP_0). As DGP under the alternative hypothesis, we use the estimated model for U.S. ($DGP_{\alpha_1 - \alpha_2 = 0.8}$). That is, respectively,

$$\Theta_0 = (0.0863, 0.2542, 0.4156, 0.023, 1.613, -0.678, -1.439, 0.031)';$$

$$\Theta_1 = (0.4, 0.12, 0.69, 1.343, 0.562, 1.237, -0.65, -1.626, 0.611)'; \quad \gamma = 0.3.$$

To study the effect of the size of the difference $\alpha_1 - \alpha_2$ on the performance of the test, we vary α_1 between (1:062; 1:562); while α_2 remains constant at its fixed value. Each of these values gives rise to $DGP_{\alpha_1 - \alpha_2 = 0.5}$ and $DGP_{\alpha_1 - \alpha_2 = 1}$, respectively.

The experiments proceed by generating artificial series of length $T + 50$ according to M_0 or M_1 with $T = 150$, and initial values set to zero.

We then discard the first 50 pseudo-data points in order to attenuate the effect of initial conditions and the remaining T points are used to compute the test statistic. We simulate the proportion of rejections of the test at the 5%; 10% and 20% significance levels. The estimation of the rejection probabilities is calculated from $B = 100$ bootstrap replications and $R = 500$ simulation runs. The processing time becomes excessive when greater values of B or R are used.

Simulation results

In Table 4 we present simulation evidence concerning the empirical size and power of the test under both RB and WB. We observe a reasonable approximation of the nominal level at all significance levels considered. Deviations from the null hypothesis are detected with high probability across the various parameterizations. We observe that in all cases under consideration the test based on the wild bootstrap approach yields slightly lower rejection probabilities than the residual bootstrap test. Thus, with homoskedastic errors, the penalty attached to using the wild bootstrap is very small. As expected, the performance of both bootstrap procedures improves as the difference between the values of parameters in the two regimes increases.

B. ESTIMATION PROCEDURES

In this appendix we present different filters that have been proposed in the relevant literature for estimating the sort of model described in Section 2. Firstly, we examine the Kalman filter, which allows us to estimate JP's model. Secondly, we introduce the threshold Kalman filter, which is a Kalman filter modified to include a threshold state equation.

The Kalman filter

In 1960, R.E. Kalman published a famous paper describing a recursive solution to the discrete data linear filtering problem. Since that time, greatly due to advances in digital computing, the Kalman filter has been the subject of extensive research and applications, particularly in the area of autonomous or assisted navigation.

The Kalman filter is a set of mathematical equations that provides an efficient recursive computational procedure for estimating the state of a process, in a way that minimizes the mean squared error (MSE)⁹. The filter is very powerful in several aspects: it supports estimations of past, present, and even future states, and it can do so even when the precise nature of the system modelled is unknown.

To start with, consider an $(n \times 1)$ vector of observed variables at date t , y_t .

These observable variables are related to a possibly unobserved $(r \times 1)$ vector h_t , known as the state vector, via a *measurement equation*,

$$(B.1) \quad y_t = H'h_t + A'x_t + w_t,$$

where H' and A' are matrices of parameters of dimensions $(n \times r)$ and $(n \times k)$, respectively; x_t is a $(k \times 1)$ vector containing exogenous or lagged dependent variables, and w_t is an $(n \times 1)$ white noise disturbance vector with $E(w_t w_t') = R$ for $t = \tau$, and 0 otherwise. Despite the fact that the variables of h_t are, in general, not observable, they are known to be generated by a first-order Markov process,

$$(B.2) \quad h_t = Fh_{t-1} + \Pi'x_t + v_t,$$

where F and Π' are matrices of parameters of dimensions $(r \times r)$ and $(r \times k)$, respectively.

⁹

The $(r \times 1)$ vector v_t is a white noise disturbance vector with $E(v_t v_t') = Q$ for $t = \tau$, and 0 otherwise. Equation (B.2) is known as the *transition equation*.

The disturbances v_t and w_t are assumed to be uncorrelated at all lags, i.e. $E(v_t w_t') = 0$ for all t and τ .

Further assumptions on measurement and transition disturbances are as follows: i) they are uncorrelated with the exogenous variables; ii) they are assumed to be normally distributed in order to calculate the likelihood function.

The state-space form that represents the dynamics of the univariate time series y_t is composed of equations (B.1) and (B.2). There are two sets of unknowns: the parameters of the model H' , A' , R , F , Π' and Q (these matrices will be referred as the system matrices), and the elements of the state vector h_t . We will assume for now that the particular numerical values of the system matrices are known. The goal of the Kalman filter procedure is to form a forecast of the unobserved state vector at time t based on the information at date $t - 1$. The information set at time $t - 1$ is given by matrix $\Psi_{t-1} \equiv (y_{t-1}', y_{t-2}', \dots, y_1', x_{t-1}', x_{t-2}', \dots, x_1')'$. Let $\hat{h}_{t|t-1}$ denote the linear forecast of the state vector h_t based on (x_t, Ψ_{t-1}) , and $P_{t|t-1}$ denote the MSE matrix associated with this forecast.

Given that the filter is a recursion, it is started assuming initial values for the mean and variance of the state variables, $\hat{h}_{1|0}$ and $P_{1|0}$, respectively. We can therefore conduct the Kalman filter in four major steps. Firstly, we calculate the one-period-ahead forecast of the unobserved state vector and its associated MSE at $t - 1$:

$$\begin{aligned}\hat{h}_{t|t-1} &= E[h_t | x_t, \Psi_{t-1}] = F \hat{h}_{t-1|t-1} + \Pi' x_t, \\ P_{t|t-1} &= E[(h_t - \hat{h}_{t|t-1})(h_t - \hat{h}_{t|t-1})' | \Psi_{t-1}] = F P_{t-1|t-1} F' + Q.\end{aligned}$$

Secondly, we calculate the one-period-ahead forecast of the measurement variable at $t - 1$:

$$(B.3) \quad \hat{y}_{t|t-1} = E(y_t | x_t, \Psi_{t-1}) = H' \hat{h}_{t|t-1} + A' x_t.$$

Thirdly, once the new observation y_t becomes available at date t , we calculate the *innovation* and the *innovation covariance matrix*:

$$(B.4) \quad \begin{aligned}\lambda_t &= y_t - \hat{y}_{t|t-1}, \\ \Lambda_t &= E[(y_t - \hat{y}_{t|t-1})(y_t - \hat{y}_{t|t-1})' | \Psi_t] = H' P_{t|t-1} H + R.\end{aligned}$$

Finally, we update the state estimate and the estimate MSE:

$$\begin{aligned}\hat{h}_{t|t} &= E[h_t | \Psi_t] = \hat{h}_{t|t-1} + \Phi_t \lambda_t, \\ P_{t|t} &= (I - \Phi_t H') P_{t|t-1},\end{aligned}$$

where $\Phi_t = P_{t|t-1} H \Lambda_t^{-1}$ is known as the *Kalman gain matrix* since a certain fraction of the difference between the observed and the predicted measurement variable is added to the previous prediction of the state vector. $\hat{h}_{t|t}$ and $P_{t|t}$ are inputs of the next filter iteration.

Hence, if the system matrices are known the Kalman filter will yield as outcome the sequences $\{\hat{h}_{t|t-1}\}_{t=1}^T$ and $\{P_{t|t-1}\}_{t=1}^T$.

We can view the Kalman filter as a sequential updating procedure that consists of forming a prior guess about the state of nature and then adding a correction to that guess, this correction being determined by how well the guess has performed in predicting the next observation. However, the state-space model is not entirely estimated since we do not usually know the parameters of the system matrices. Assuming that $\{v_t, w_t\}_{t=1}^T$ are normally distributed, then the

distribution of y_t conditional on (x_t, Ψ_{t-1}) is Normal with mean given by (B.3) and variance given by (B.4). We use the *prediction error decomposition* to construct the logarithm of the distribution function as follows:

$$\ln f(y_t|x_t, \Psi_{t-1}) = -\frac{T}{2} \ln 2\pi - \frac{1}{2} \ln |\Lambda_t| - \frac{1}{2} \lambda_t' \Lambda_t^{-1} \lambda_t.$$

To estimate the parameters of the system matrices, we maximize the log-likelihood function $\ln L = \sum_{t=1}^T \ln f(y_t|x_t, \Psi_{t-1})$ with respect to the underlying unknown parameters using nonlinear optimization techniques.

The threshold Kalman filter

Nonlinearities can be introduced into state-space models in a variety of ways. One of the most important classes of models has Gaussian (or Normal) disturbances but allows the system matrices to depend on past observations available at time $t - 1$. This class of models is known in time series literature as *conditionally Gaussian*¹⁰. These models have the attractive property of still being tractable by the Kalman filter. In our model, we only introduce regime-switching in the state equation. The state-space representation is the following:

$$\begin{aligned} y_t &= H'h_t + A'x_t + w_t \\ h_t &= F(q_{t-1})h_{t-1} + \Pi'x_t + v_t, \end{aligned}$$

where q_{t-1} stands for a stationary threshold variable. Despite the fact that the coefficient matrix associated with h_{t-1}

depends on observations up to and including $t - 1$, it may be regarded as being fixed once we are at time $t - 1$. The same hypotheses about the disturbance vectors v_t and w_t are retained.

Hence the derivation of the Kalman filter proceeds as in the previous section but a simple modification is introduced. As mentioned above, the goal of the Kalman filter procedure is to derive a forecast of the unobserved state vector h_t based on the information set Ψ_{t-1} .

Here the goal is to form a forecast of h_t conditional not only on (x_t, Ψ_{t-1}) but also on the regime of the economy. Let j be a dummy variable that refers to the regime of the economy, i.e. $j = 1$ if $q_{t-1} \geq \gamma$, and $j = 2$ if $q_{t-1} < \gamma$.

We calculate the conditional forecast of the state variables and its conditional error covariance, or MSE, matrix as follows:

$$\begin{aligned} \hat{h}_{t|t-1}^j &= F_j \hat{h}_{t-1|t-1}^j + \Pi'x_t \\ P_{t|t-1}^j &= F_j P_{t-1|t-1}^j F_j' + Q, \end{aligned}$$

where F_j refers to the transition matrix in each regime.

The conditional forecast of observed variables is given by:

$$\hat{y}_{t|t-1}^j = H' \hat{h}_{t|t-1}^j + A'x_t.$$

¹⁰See Harvey (1989, Section 3.7.) for a more detailed description of this class of models.

Once observable variables are realized at date t , we can calculate the conditional error forecast and its conditional variance:

$$\begin{aligned}\lambda_t^j &= y_t - \hat{y}_{t|t-1}^j \\ \Lambda_t^j &= H' P_{t|t-1}^j H + R.\end{aligned}$$

Finally, we update the previous conditional forecast of unobserved variables and its conditional variance as follows:

$$\begin{aligned}\hat{h}_{t|t}^j &= \hat{h}_{t|t-1}^j + \Phi_t^j \lambda_t^j \\ P_{t|t}^j &= (I - \Phi_t^j H') P_{t|t-1}^j,\end{aligned}$$

with $\Phi_t^j = P_{t|t-1}^j H (\Lambda_t^j)^{-1}$. These last two terms correspond to the inputs of the next filter iteration.

In our particular case, $q_{t-1} = U_{t-1} - U_{t-d}$. To estimate parameters γ and d we first construct a grid $\Delta = \Theta \otimes \Pi$ over the two-dimensional space (γ, d) , where Θ and Π are the grids for γ and d , respectively.

We proceed in two steps. Firstly, we estimate the model for each candidate (γ, d) belonging to the selected grid. That is, conditionally on (γ, d) we maximize the log-likelihood function $\ln L(\gamma, d) = \sum_{t=1}^T \ln f(y_t | x_t, \Psi_{t-1}, \gamma, d)$ with respect to the underlying unknown parameters using nonlinear optimization techniques. Secondly, we retain the values of the threshold parameter and the delay lag that provide the highest log-likelihood. That is, $\hat{\gamma}$ and \hat{d} are given by:

$$(\hat{\gamma}, \hat{d}) = \arg \max_{(\gamma, d) \in \Delta} \ln L(\gamma, d).$$

C. TABLES AND FIGURES

Table 1: Unit Root Tests^a

<i>p</i> -value of ADF test on GDP series	
Italy	0.9970
France	0.2622
U.S.	0.9459
<i>p</i> -value of Caner and Hansen's test on Unemployment series	
Italy	0.002
France	0.001
U.S.	0.000

^a For the ADF test, we use Mackinnon (1996) one-sided *p*-values for the null hypothesis of a unit root. For the test of Caner and Hansen (2001), the relevant null hypothesis is that there is not a unit root.

Table 2 : Tests for the Hysteresis Assumption

	Nonlinear Model $H_0 : \alpha_1 = \alpha_2$	Linear Model $H_0^{JP} : \alpha = 0$
Italy	Bootstrap p -value=0.0021	p -value=0.269
France	Bootstrap p -value=0.0015	p -value=0.285
U.S.	Bootstrap p -value=0.0013	p -value=0.405

Table 3 : Estimation Results for the Nonlinear Model^a

	ITALY		FRANCE		U.S.	
Natural Rate Eq.						
Regime	$i = 1$	$i = 2$	$i = 1$	$i = 2$	$i = 1$	$i = 2$
α_i	2.512 (0.101)	1.476 (0.042)	3.540 (0.913)	1.570 (0.022)	1.343 (0.121)	0.562 (0.022)
σ_N	0.431 (0.021)		0.416 (0.035)		0.400 (0.018)	
Cyclical Rate Eq.						
ϕ_1	1.521 (0.187)		1.615 (0.041)		1.237 (0.082)	
ϕ_2	-0.638 (0.050)		-0.733 (0.056)		-0.650 (0.089)	
σ_C	0.020 (0.004)		0.139 (0.074)		0.120 (0.023)	
Identification Eq.						
β	0.463 (0.023)		0.600 (0.069)		0.611 (0.042)	
δ	-5.401 (1.211)		-0.390 (0.052)		-1.626 (0.0159)	
σ_D	0.640 (0.035)		0.540 (0.033)		0.690 (0.031)	
Threshold	$\gamma = 0.1$		$\gamma = 0.16$		$\gamma = 0.3$	
90% - CI ^b	[0.023, 0.247]		[0.123, 0.532]		[0.143, 0.632]	
Delay lag	$d = 2$		$d = 3$		$d = 2$	
% observations	41%	59%	44%	56%	42%	58%

^a Following Stofer and Wall (1991), standard errors are calculated from $B = 1000$ runs of the bootstrap and provided into brackets. These standard errors are the square root of $\sum_{b=1}^B (\hat{\theta}_b^* - \hat{\theta}_i)^2 / (B - 1)$, where $\hat{\theta}_i$ represents the i th parameter of the vector Θ_1 , $i = 1, \dots, 9$, and $\hat{\theta}_i$ is the ML estimate of θ_i .

^b We compute the confidence interval based on the bootstrap percentiles described by Hall (1992).

Table 4: Monte Carlo Results

Nominal size		5%	10%	20%
Simulated size				
DGP_0	RB	0.051	0.096	0.196
	WB	0.042	0.093	0.203
Simulated power				
$DGP_{\alpha_1 - \alpha_2 = 0.5}$	RB	0.566	0.587	0.590
	WB	0.455	0.520	0.549
$DGP_{\alpha_1 - \alpha_2 = 0.8}$	RB	0.710	0.723	0.741
	WB	0.649	0.690	0.703
$DGP_{\alpha_1 - \alpha_2 = 1}$	RB	0.899	0.930	0.976
	WB	0.795	0.887	0.900

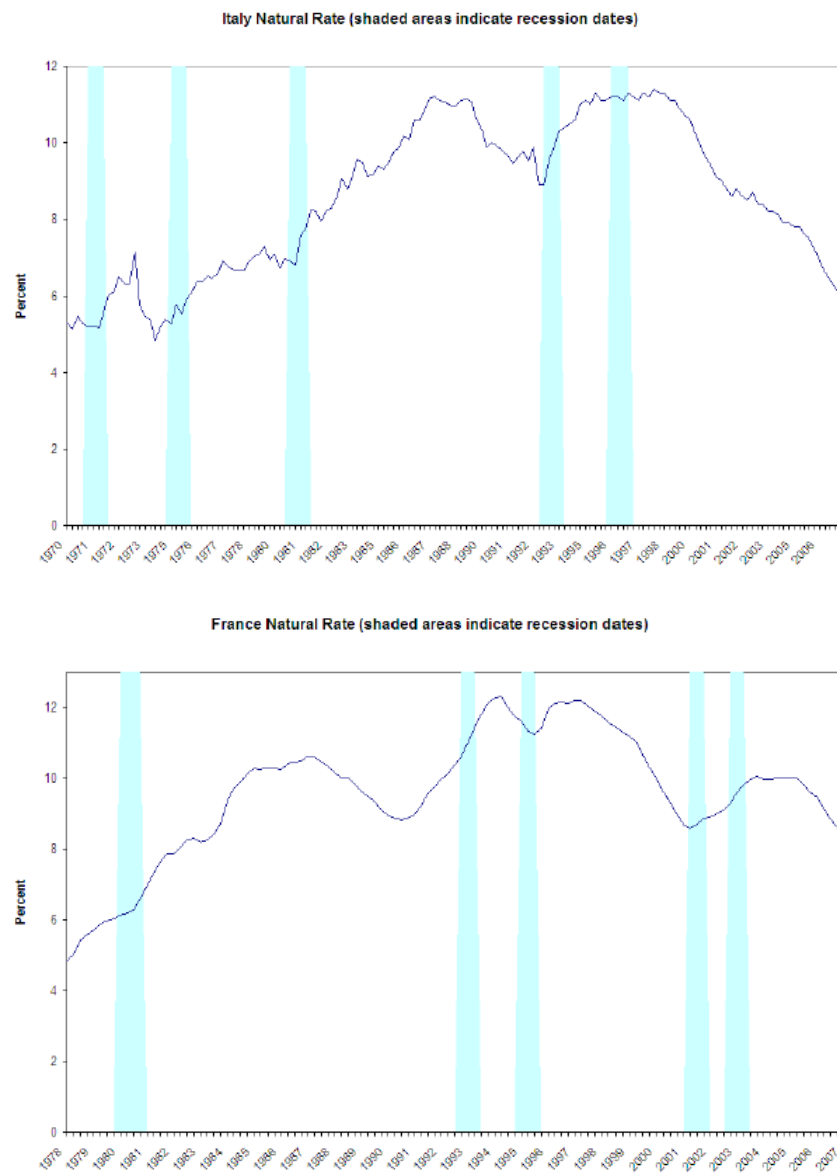


Figure 2: France

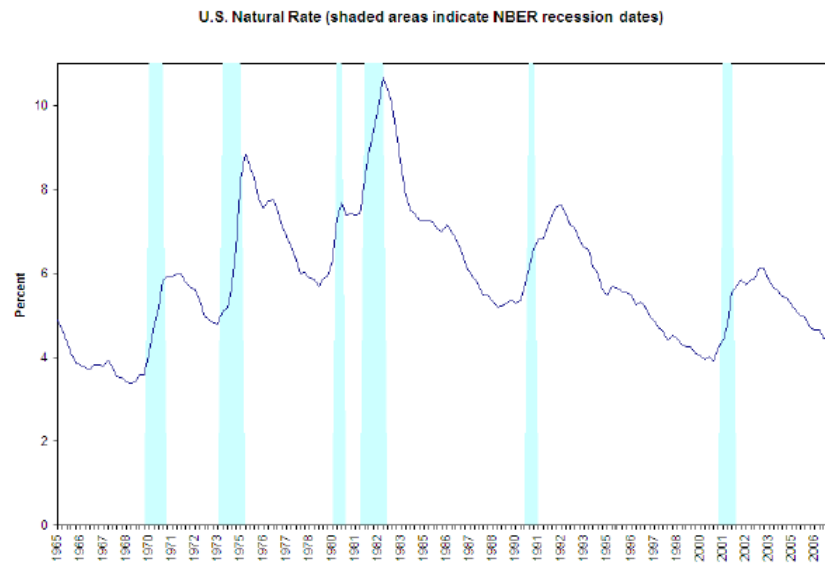


Figure 3: U.S.

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Ministry of Economy and Finance

Department of the Treasury

Directorate I: Economic and Financial Analysis

Address:

Via XX Settembre, 97
00187 - Rome

Websites:

www.mef.gov.it
www.dt.tesoro.it

e-mail:

dt.segreteria.direzione1@tesoro.it

Telephone:

+39 06 47614202
+39 06 47614197

Fax:

+39 06 47821886